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Discontinuous Control Analysis and Design for Temperature Control of Thermal System

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Abstract*— Temperature control system is a widely applied process since temperature plays a major role in our life starting from room temperature conditioning to various industrial and medical applications. In this paper, a control algorithm is proposed for controlling the temperature of a certain process. Analysis is performed to verify the feasibility of the proposed control algorithm. A simulation is performed using MATLAB software to show the performance of the proposed control algorithm and a practical implementation is performed using Arduino to investigate the validity of the analysis. The results show the ability of the proposed controller to achieve the desired results which confirms the validity of the proposed controller and mathematical analysis.*

Index Terms*— temperature control system, discontinuous control, stability analysis.*

I. INTRODUCTION

Temperature control refers to the processes that are aimed at maintaining the temperature in a given area at certain maximum/minimum level or within a certain range. This process is commonly used in most areas of the world. Recently, globalization and industrialization has further necessitated the need for Temperature Control applications in various daily activities, especially with the advent of the greenhouse effect [1].

Many Homes and Industries among other areas maintain certain sections of operation that must be maintained within a certain temperature for process to work successfully. In research laboratories, the lack of use of Temperature Control Systems has lead to the purchase of chambers of various sizes where temperature specific research work would be kept. This has also lead to an increase in overhead cost. In areas that have electronic activities or machinery functioning constantly, such as in server rooms and production plants. These are places where heavy machinery and computers work continuously 24 hours every day [2,3]. During these processes, the temperature needs to be monitored frequently in order to ensure that it doesn’t rise or fall below a value that would accelerate wearing out of the systems. It is important also to monitor the level of temperature various other places such as morgues, hospitals, aircrafts, living rooms, etc, to ensure that thermal comfort is maintained [4,5].

Automatic temperature control is certified as the best method in any application because the temperature is usually controlled automatically (no human intervention involved) throughout the process. The results obtained from various applications of the process across different regions and timelines shows the temperature is controlled effectively and more accurately. In addition, this finding also makes human work easier as an automatically controlled system worries about other contingent weather issues for you. Due to its slow dynamics and comparatively simpler model, simple control methods have been applied to the temperature control to reduce costs [6,7].

In general, thermal physical systems are considered as slow processes and their models are commonly described using first order models. For that reason, it becomes cost inefficient to implement complicated control schemes and high precision temperature control can be achieved using less

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sophisticated control algorithms and even an on/off mechanisms can achieve the desired performance [8, 9, 10].

A low-cost temperature controlling system on paper to be used in paper-based microfluidics was implemented in [11] which utilized PID controller in a closed-loop system. An accuracy of temperature is below 0.22 C° was achieved which is suitable for biological temperature-sensitive applications. A closed loop temperature control system design using conventional and advanced intelligent controller was investigated in [12] where conventional Proportional Integral (PI) controller as well as modern Artificial Neural Network (ANN) based PI controller have been used as the control strategies when a model of PI controller based on ANN with Levenberg-Marquardt learning algorithm has been introduced. The simulation report of projected design has been shown and it has been concluded that the proposed control strategy is very advantageous due to its small overshoot, fast response, and high precision.

An experimental work on temperature control using on-off algorithm combined with PID algorithm was introduced in [13] where the tuning parameters of the PI controller are calculated using the Ziegler-Nichols criteria, available for the slow processes with one time constant and delay time. In turn, these parameters are obtained by offline identification, using the Cohen-Coon graphical method. A temperature control system using conventional PID and intelligent fuzzy logic controller was designed in [14]. it has been found that the output response from Fuzzy Logic is very accurate in terms of overshoot and steady state error when compared to that of PID.

An analysis and design of temperature control system using conventional PI and advanced ANN controllers was carried on in [15]. Firstly conventional Proportional Integral (PI) controller has been used for the purpose and furthermore Artificial Neural Network (ANN) based PI controller has improved the whole system’s transient response.

The main contribution of this work is to mathematically find the suitable criteria for the temperature control of a certain thermal system and prove the validity of the proposed analysis and control.

In this paper, a discontinuous control algorithm have been proposed for the temperature control system. The mathematical model for a certain thermal system have been introduced in section 2 and a control algorithm have been proposed to control the temperature of the thermal system in section 3. Analysis have been made to investigate the validity of the control algorithm and a software simulation has been performed to show the validity of the analysis. A software simulation for the system and control is made to show the performance of the control algorithm in section 4 and some conclusions and remarks were mentioned in section 5.

II. MATHEMATICAL MODELING OF THERMAL SYSTEM

Consider the thermal system shown in *Fig. 1* below. The system consists of a tank that contains a sort of liquid at a temperature T_i (C^o). The tank is thermally insulated from the outer environment at temperature T_o (C^o) with a thermal insulator with thermal resistance R (m·K/W). The heat is introduced to the liquid inside the tank through a heater with a heat flow q_i (W). The thermal power lost to the outer environment is q_o (W).

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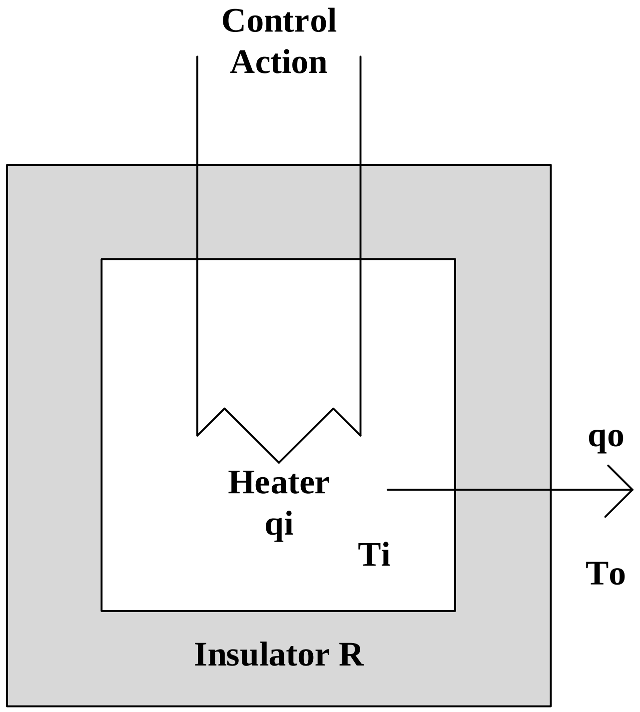

FIG. 1. THERMAL INSULATED SYSTEM.

Considering the above parameters, the following model can be derived as follows: the heat flow to the outer environment can be written as the difference in temperature divided by the thermal resistance R as follows: the heat flow to the outer environment can be written as the difference in temperature divided by the thermal resistance R as follows

$$q_o = \frac{1}{R}(T_i - T_o) \quad (Watt) \tag{1}$$

Consider that the liquid inside the tank have a total thermal capacitance C (J/K), then the heat content of the liquid inside the tank Q (J) can be written as

$$Q = CT_i \qquad (Joule) \tag{2}$$

Where

$$C = C_o \cdot m \qquad (Joule/K)$$

Where C_o is the specific heat capacity of substance (J/(K·kg)) and m denotes the mass of substance. Derive Eq. (2) wrt. results in

$$\frac{dQ}{dt} = C\frac{dT_i}{dt} \quad (Watt) \tag{3}$$

Eq. (3) above shows that the rate of change of the amount of internal energy of the liquid is proportional to the rate of change of the liquid temperature, assuming fixed thermal capacitance (C).

In the thermal system mentioned above, it’s obvious to conclude that the change of the liquid energy is the difference between the input power to the liquid and the heat losses through the insulator to the outer environment. That’s the above statements can be written as

$$\frac{dQ}{dt} = q_i - q_o \tag{4}$$

Combining Eq.’s (3) and (4) results in

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$$C\frac{dT_i}{dt} = q_i - q_o \tag{5}$$

Substitute for q_o from Eq. (1) into the above equation results in

$$C\frac{dT_i}{dt} = q_i - \frac{1}{R}(T_i - T_o) \tag{6}$$

Rearranging the terms in the above equation results in

$$\frac{dT_i}{dt} = \frac{1}{C}q_i - \frac{1}{RC}T_i + \frac{1}{RC}T_o \qquad (Kelvin/sec) \tag{7}$$

The equation above describes the mathematical model of the thermal system and will be used in the following analysis.

III. DISCONTINUOUS CONTROL DESIGN

Consider that the heat flow introduced into the tank is controlled by an on/off switch, then Eq. (7) can be written as

$$\frac{dT_i}{dt} = \frac{1}{C}uq_i - \frac{1}{RC}T_i + \frac{1}{RC}T_o \tag{8}$$

Where u is the control action defined in Eq. (9) below

$$u = \begin{cases} 1 & (on) \\ 0 & (off) \end{cases} \tag{9}$$

The first step in control design is to define the temperature error e as

$$e = T_i - T_d \tag{10}$$

In this paper, it’s desired to design a control law u(t) that stabilizes the temperature of the liquid inside the tank (T_i) at a desired temperature value (T_d), that’s:

$$T_i = T_d \quad \rightarrow \quad e = 0$$

From the statements above, then the main aim of the controller is to enforce the value of the temperature error e to reach zero value which results in the desired requirements.

In order to design a suitable control action u that satisfies the above requirements, the stability theory is applied as follows [16,17]: first, define a positive definite Lyapunov function candidate as

$$V(e) = \frac{1}{2}e^2 \quad > 0 \quad \forall e \neq 0 \tag{11}$$

Derive the above equation wrt.

$$\dot{V}(e) = e\dot{e}$$

According to the Lyapunov stability theory, in order for the error value e to reach zero value, then the time derivative of the Lyapunov function should be negative definite, that’s

$$\dot{V}(e) = e\dot{e} \quad < 0 \quad \forall e \neq 0 \tag{12}$$

In order to obtain $\dot{e}$, derive Eq. (10) wrt results in

$$\dot{e} = \frac{dT_i}{dt} - \frac{dT_d}{dt}$$

In this paper, its considered that the desired temperature value T_d is fixed which means that

$$\frac{dT_d}{dt} = 0$$

Which results in

$$\dot{e} = \frac{dT_i}{dt}$$

Substitute for $\frac{dT_i}{dt}$ from Eq. (8) into the above equation results in

$$\dot{e} = \frac{1}{C}uq_i - \frac{1}{RC}T_i + \frac{1}{RC}T_o \tag{13}$$

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Substitute the above equation into Eq. (12), then Eq. (12) can be written as

$$\dot{V}(e) = e\left[\frac{1}{C}uq_i - \frac{1}{RC}T_i + \frac{1}{RC}T_o\right] \tag{14}$$

In this work, its assumed that the heat flow to the tank is controlled by an on/off switch defined by the discontinuous function below

$$u = \frac{1}{2}\big(1 - sign(e)\big) = \begin{cases} 1 \;\; (on) & if\ e < 0 \quad (T_i < T_d) \\ 0 \;\; (off) & if\ e > 0 \quad (T_i > T_d) \end{cases} \tag{15}$$

The equation above defines two regions for the operation of the system, one is for u = 1 (on) and the other is for u = 0 (off).

For the first case when $(T_i < T_d) \rightarrow (e < 0)$, then u = 1 (on). Since $(e < 0)$, then it's required that $(\dot{e} > 0)$ which results in

$$\frac{1}{C}q_i - \frac{1}{RC}T_i + \frac{1}{RC}T_o > 0 \tag{16}$$

Multiply by RC

$$Rq_i - T_i + T_o > 0$$

From the above equation one can find that

$$q_i > \frac{T_i - T_o}{R} \tag{17}$$

Eq. (17) shows that, in order for the control action to be able to stabilize the temperature, the heat flow to the tank q_i should satisfy the inequality of Eq. (17).

On the other hand, when $(T_i > T_d) \rightarrow (e > 0)$ then u = 0 (off). In this case, since $(e > 0)$, then it's required that $(\dot{e} < 0)$ which results in

$$-\frac{1}{RC}T_i + \frac{1}{RC}T_o < 0 \tag{18}$$

Multiply by RC

$$-T_i + T_o < 0$$

Which results in

$$T_i > T_o \tag{19}$$

Eq. (19) simply means that, in order for the temperature to reach the desired value, the inner tank temperature should higher than the outer temperature where heat loss causes the tank temperature to drop. In the following discussion, a software simulation is performed to show the validity of the analysis above.

A block diagram of the complete system is shown in *Fig. 2* below.

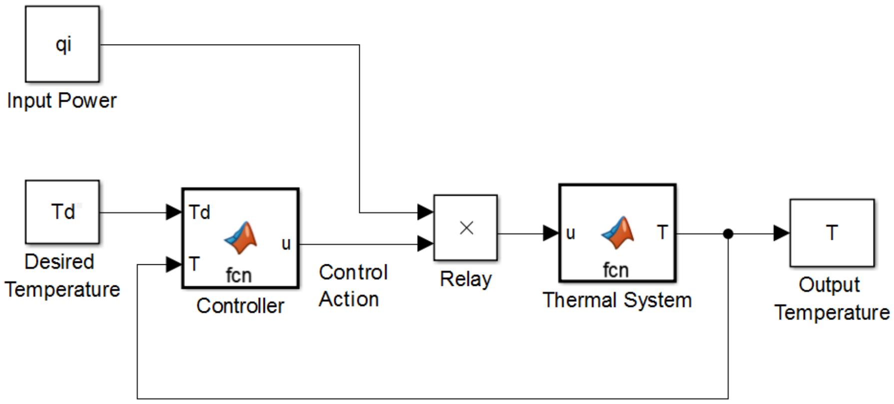

FIG. 2. COMPLETE BLOCK DIAGRAM OF TEMPERATURE SYSTEM.

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From the discussion in section 3 above, it can be seen from inequalities (17) and (19) that the stability is guaranteed whenever these conditions are fulfilled.

IV. SIMULATION

In order to investigate the performance of the proposed control system, a software simulation is performed using MATLAB. The simulation parameters were selected to simulate a real-life water heating system used in conventional home appliance. Its assumed that the tank is of cylindrical shape filled with water with 0.4 m diameter and 1.2 m height. Also, its assumed that the tank is thermally insulated by a 1 cm of fiber glass. According to the above values, the tank volume will be equal to 150 L and the liquid mass will be 150 kg respectively.

The thermal resistance can be calculated according to the formula below:

$$R = \rho \frac{l}{A}$$

Where ρ is the specific thermal resistivity (m·K/W), l is the insulator thickness (cm) and A is the external area of the tank (m^2). For the fiber glass insulator considered in this work, the thermal resistivity (ρ) has the value

$$\rho = 25\ (m \cdot K/W).$$

The tank external area is calculated as

$$A = 0.4 \cdot \pi \cdot 1.2 + 2(\pi \cdot 0.2^2) = 1.759\ m^2$$

Then, the total thermal resistance for the tank insulator is

$$R = \rho \frac{l}{A} = 25 \times \frac{0.01}{1.759} = 0.142\ \ (K/W)$$

Also, the thermal capacitance of the tank is calculated as

$$C = m \cdot C_o = 150 \cdot 4200 = 630000\ (J/K)$$

Where m = 150 kg and $C_o = 4200\ \big(J/(K \cdot kg)\big)$

The tank supposed to be heated using a 3 kW electrical heater, that's

$$q_i = 3\ kW$$

Using the values above in simulation model, the results are obtained.

In the simulation, its desired to heat and maintain the liquid temperature at 75 C^o, while temperature out of the tank is 25 C^o, i.e. $T_d = 75$ and $T_o = 25\ C^o$

According to the values mentioned previously, the condition of inequality (17) can be found as

$$(q_i = 3000\ W) > \left(\frac{T_d - T_o}{R} = \frac{75 - 25}{25} = 2\ W\right)$$

That's, the control action should able to heat and stabilize the tank temperature at the desired level. The simulation results are shown in *Fig's. 3-5* below. *Fig. 3* shows the tank temperature. *Fig. 4* shows the temperature error and *Fig. 5* shows the control action.

From *Fig. 3* below, it can be seen that how the temperature of the liquid inside the tank starts from an initial temperature value of 25 C^o and how it rises due to the control action until it reaches the desired value of 75 C^o. Then, the control action works to stabilize the temperature at that level.

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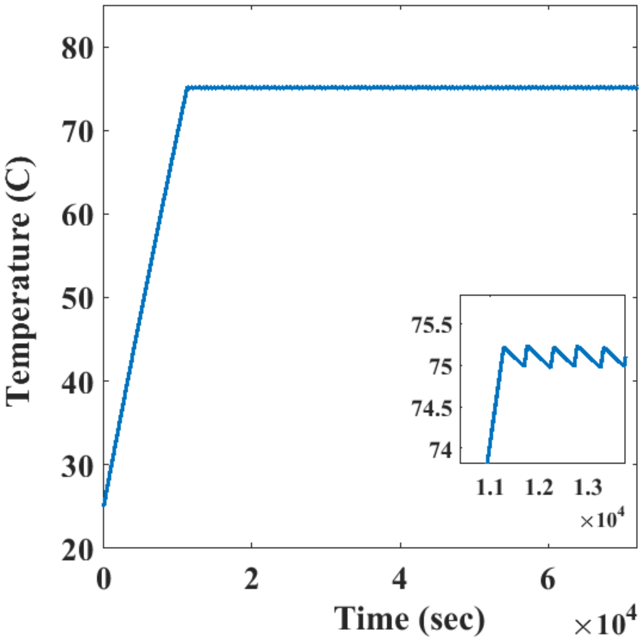

FIG. 3. TANK TEMPERATURE.

In *Fig. 4* the control action is shown. It can be seen how the control action remains on for the duration from the start of operation until the system reaches the desired temperature. Then it chatters between it's on and off states in order to keep the liquid inside the tank at the desired temperature. This chatter is due to the heat losses through the thermal insulator to the outer environment which requires the control action to work constantly to rectify the temperature deviation from the desired temperature.

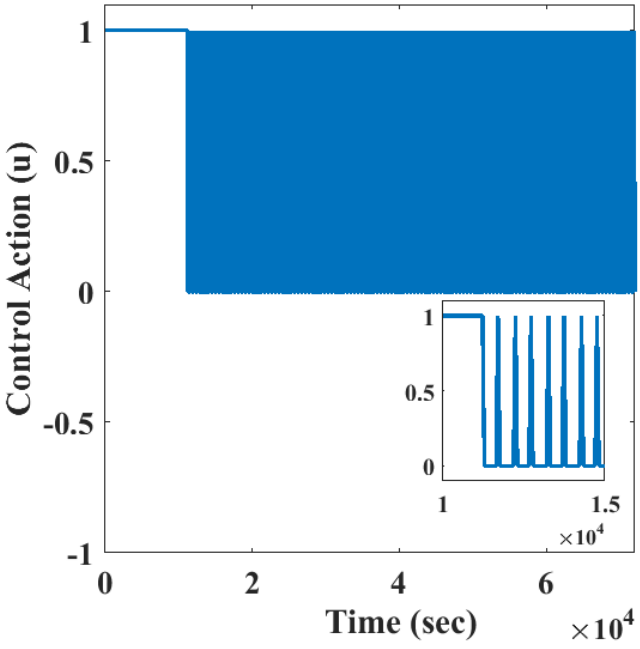

FIG. 4. CONTROL ACTION.

Fig. 5 below shows the temperature error. It can be noted how the error starts from an initial value of -50 then it reaches zero value as the liquid temperature reaches the desired value.

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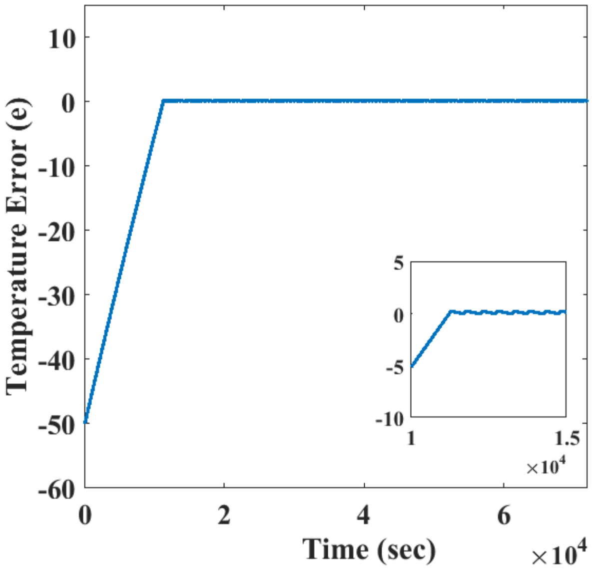

FIG. 5. TEMPERATURE ERROR.

A final plot is added for the root mean square (RMS) value of the error in *Fig. 6* below. The RMS value gives an indicator for the system performance.

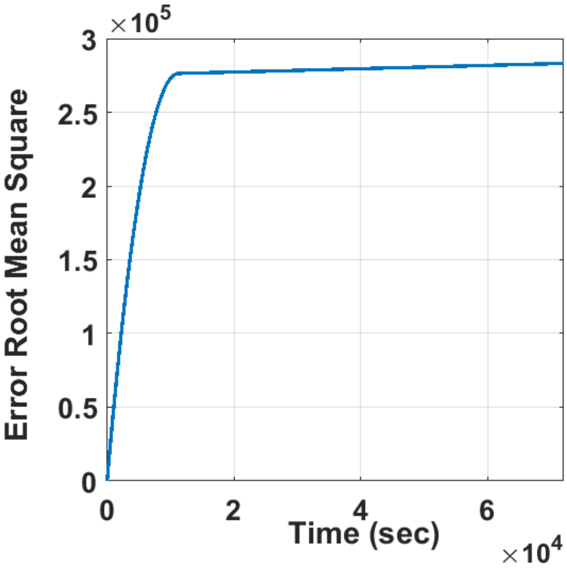

FIG. 6. ERROR RMS.

A quantitative evaluation of the simulated results is shown Table I below:

TABLE I. TYPE SIZES AND APPEARANCE

Parameter	**Value**
Reaching Time	11200 sec (3 hrs 5 min)
Peak overshoot	no
Peak time	no
Settling time	11200 sec (3 hrs 5 min)

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A future work can be added to the proposed system. One of them is to use a more sophisticated control algorithm for more precise control and also for less chattering switching. However, applying more complicated algorithms might come at the cost of more implementation complexity and high costs of additional components.

V. CONCLUSIONS

In this paper, a temperature control system has been analyzed. The mathematical has been introduced and a discontinuous control mechanism was proposed for the control of the thermal system. Analysis were carried to investigate the ability of the proposed control to stabilize the temperature of the system at a desired value and a software simulation was performed to show the validity of the analysis. The results shows that the proposed control was able to maintain the system temperature at the desired value.

It can be seen from the results how the the proposed controller succeeds in controlling the liquid temperature where it reaches the desired temperature within 3 hours (11×10^4 sec) dependin on the liquid mass and heater input power. Also, it canbe noted how the control action chatters between its on and off states due to its discontinuous nature. The reason behind choosing such a discontinuous control action is that, firstly, its the most common control action used in many temperature control applications. The reason behind that is, first, most temperature control applications consumes a considerable amount of energy making on/off actuators more suitable for such applications due to its low cost and also for its simplicity which reduces the implementation cost and complexity. Another reason also is that temperature control systems in general are considered as s slow processes which have a large time constants which means that even a simple control action as the one proposed in this work can work efficiently in controlling the system.

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