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Quantitative PID Controller Design using Black Hole Optimization for Ball and Beam System

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Abstract*— In this work, the design of a quantitative PID controller is proposed for Ball and Beam system. This controller is designed to robustly compensate for the nonlinear and uncertain behavior of the ball and beam system. The PID controller parameters are obtained using the Black Hole Optimization (BHO) method subject to Quantitative Feedback Theory (QFT) consteaints. The QFT is used to design a simple and robust controller with a more desirable performance. The simulation results show that the proposed quantitative PID controller can effectively compensate the ball and beam system with robust behavior and desirable time response specifications.*

Index Terms*— Robust control, Quantitative Feedback Theory (QFT), PID, Ball and Beam System, Black Hole Optimization.*

I. INTRODUCTION

Quantitative feedback theory is a frequency domain approach for achieving the desired robust specifications. It was built by Horowitz in (1960). The idea of QFT is to translate the design specifications of a closed-loop system and plant uncertainties to robust stability and performance bounds to design the controller by loop shaping techniques. In systems with parametric uncertainty models, plant templates must be produced in the first place at a fixed frequency. On the nominal open-loop function, QFT transforms closed-loop specifications into magnitude and step constraints. The nominal open-loop feature is then configured to meet its specifications and maintain nominal loop stability. After the loop has been closed, a prefilter will be built [1].

On the other hand, the PID controller is the most commonly used controller. About 90 to 95% of all control problems are solved by the PID controller. This controller can be used in different forms and tuned by different methods. Moreover, the PID controller is one of the fixed structure controllers whose structure is selected independently of the plant order [2]. A range of design methods have been used to control the ball and beam system. Prasad et al. [3] in 2014 proposed an optimal PID controller for ball and beam system. The parameters of the PID controller were tuned by the Genetic algorithm and the Differential Evolution algorithm. It was found that the PID tuned by the Differential Evolution algorithm was faster than that tuned by the Genetic algorithm.

Malini et al. [4] in 2016 proposed a PID controller to control the ball and beam system. Anjali et al. [5] in 2016 proposed a PID controller tuned by a Genetic algorithm to control ball and beam system. Ali et al. [6] in 2018 proposed a PID controller for ball and beam, where the two controllers were used to control the ball position and one for control the motor angle. The parameters of the PID controller tuned by two methods particle swarm and trial and error. It was found that the performance of the controller tuned by particle was better than the controller tuned by trial and error. Fuzzy PID controller and PID controller

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were applied by Aziz et al. [7]. It was found that the fuzzy PID controller can give better than the PID controller for steady-state error reduction and overshoot elimination. In contrast, the performance PID controller was better than the fuzzy PID in the rise time reduction.

On the other hand, the QFT and controller has been used in many applications to give robust specifications [8]. Khan et al. [9] proposed the QFT for a Grain dryer plant for controlling the drying process.

In this paper, the automatic design of a quantitative PID controller using a black hole optimization is proposed. The BHO is used to automate the loop shaping procedure and obtain optimal parameters for the PID controller subject to QFT constraints.

II. BALL AND BEAM SYSTEM MODEL

The ball and beam system is also called balancing a ball on a beam. It can be found in most university control laboratories. It is two-level arms that support the beam on both ends. One of the level arms is pinned, while the other is attached to the engine output gear. It is unstable in an open-loop system with a high nonlinearity [10]. The ball and beam system is given in *Fig.* 1.

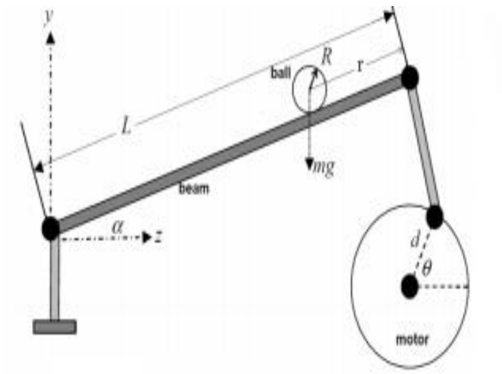

FIG. 1. THE BALL AND BEAM SYSTEM STRUCTURE

As shown in Figure 1, There are two degrees of freedom, the ball that rolls on the beam up and down, and the other is the beam that rotates around the central axis.

The Lagrangian function of movement derives the dynamic equation of the Ball and Beam system for the ball as [11]:

$$\left(\frac{J}{R^2} + m\right)\ddot{r} + mgsin\alpha - mr\dot{\alpha}^2 = 0 \quad (1)$$

where m the mass of the ball, J is the ball's moment inertia, R is the radius of the ball, g is gravitational acceleration, r is the ball position, $\ddot{r}$ is the acceleration of the ball, and α is the beam angle.

Linearize the equation (1) about beam angle $\alpha=0$, obtain as follow:

$$\left(\frac{J}{R^2} + m\right)\ddot{r} = -mg\alpha \quad (2)$$

The equation that relates the beam angle to the angle of the gear can be approximated as linear by equating the arc distance, and it is defined as:

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$$\frac{\alpha(s)}{\theta(s)} \cong \frac{d}{L} \tag{3}$$

where *d* is the level arm offset, *L* is the length of the beam, and *θ(s)* is the gear angle.
After substituting (3) in (2), obtained as follow:

$$\left(\frac{J}{R^2} + m\right)\ddot{r} = -mg\frac{d}{L}\theta \tag{4}$$

Tacking Laplace transforms, obtain as follow:

$$\left(\frac{J}{R^2} + m\right)r(s)s^2 = -mg\frac{d}{L}\theta \tag{5}$$

The transfer function of the ball position *r(s)* to the gear angle *θ(s)* is given as:

$$\frac{r(s)}{\theta(s)} = \frac{-mgd}{L(m+\frac{J}{R^2})s^2} \tag{6}$$

Table 1 contains the values of the system parameters.

TABLE 1. PARAMETERS FOR BALL AND BEAM SYSTEM [12]

Parameters for ball and beam system	**Value**
Mass of the ball	0.11 *kg*
The radius of the ball	0.015 *m*
Level arm offset	0.04 *m*
Gravitational acceleration	9.8 ms^{-2}
Length of the beam	40 *cm*
Ball's moment inertia	9.99e-6

III. BLACK HOLE OPTIMIZATION (BHO) METHOD

The principle of a black hole is essentially a field of space that concentrated so much mass that something that falls into a black hole is forever gone. In the algorithm, the best candidates are like a black hole. After the startup, the black hole continues to pull starts. If a star enters the black, a new star is spontaneously present (candidate solution). Set and create in the search area and begins a new search. The advantage of this algorithm is free from tuning parameter and to solve multi-objective optimization [13]. The method of optimization is implemented as follow[14,15,16]:

1. Initial population of candidate solutions are randomly generated.

2. Check for overall population fitness as following :

$$Z_{BH} = \sum_{i=1}^{pop\_size} eval(population(t)) \tag{7}$$

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where Z_{BH} is the fitness value of Black Hole, pop_size is population size, i is a star in the initial population and $eval(population(t))$ *represents the evaluation of populatin*.

3. The first Black Hole and stars, when initialized Black Hole continues to swallow the stars.

4. Stars moved to the black hole as in the following equations.

$$x_i(t) = x_i(t) + rand \times (x_{BH}(t) - x_i(t)) \quad (8)$$

where $x_i(t)$ is the location of star and $x_{BH}(t)$ is the location of a black hole.

5. All the stars are moved; horizon's radius of Black Hole can be given as follows :

$$c = \frac{Z_{BH}}{\sum_i^N z_i} \quad (9)$$

where Z_{BH} is fitness value of black hole, z_i is fitness value of star, and N is a number of star solution.

6. If the difference between the solution of the candidate and the Black Hole is less than c, the candidate will collapse, and the new candidate will be created and distributed in search space randomly.

7. Replace it in a random position with a new star.

8. When the fitness value reaches the final iteration, the loop will be ended.

The flowchart of the (BHO) is show in *Fig. 2*.

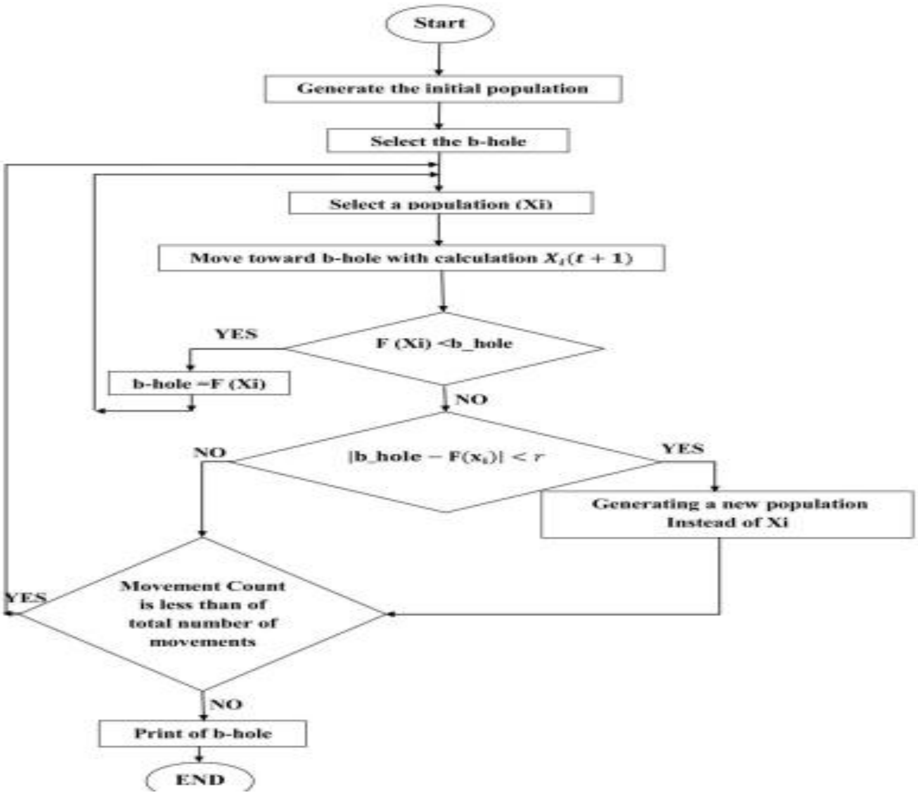

FIG. 2 FLOWCHART OF BLACK HOLE OPTIMIZATION

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IV. TUNING PID CONTROLLER USING BLACK HOLE OPTIMIZATION

The black hole optimization is used to tune the parameters of the PID controller. A good set of PID controller parameters can yield a good system response. The controller and prefilter structures are applied as follow:

$$G_c(s) = k_p + \frac{K_i}{s} + k_d s \tag{10}$$

$$f(s) = \frac{1}{(\tau_1 s+1)(\tau_2 s+1)} \tag{11}$$

where k_p, k_i *and* k_d are the PID controller parameters, τ_1 *and* τ_2 represent the time constants of the proposed prefilter.

The following settings of the (BHO) used to carry out the suggested controller design are:

1. The number of dimensions equals 5 ($k_p, k_i, k_d, \tau_1, \tau_2$).
2. Initial population equals 50.
3. The maximum number of iteration equals 35.

V. CONTROLLER DESIGN

A huge number of constraints can be quickly handled by BHO, and the first step in creating a more complex controller can use a black hole. The purpose of the PID controller tuning is to find parameters that meet the closed-loop system performance specifications and improve the control loop's robust stability subject to QFT. *Fig.* 3 shows the overall block diagram of the proposed controlled system.

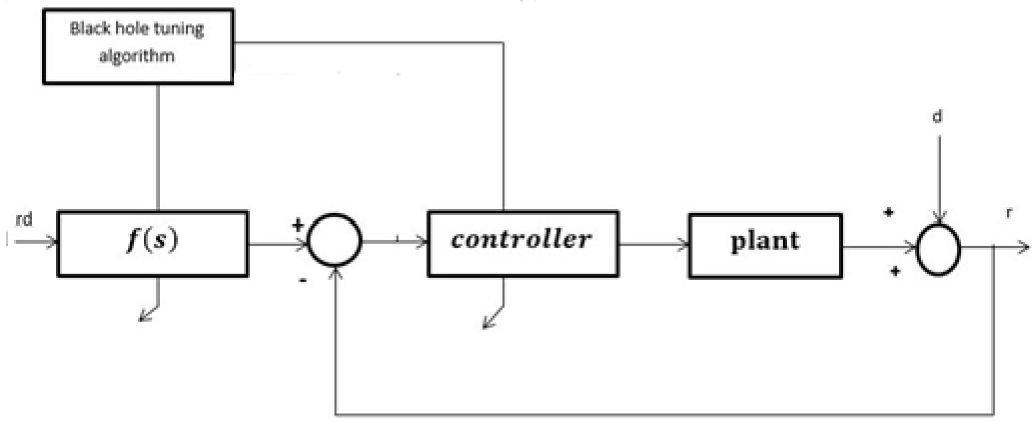

FIG. 3. BLOCK DIAGRAM OF THE PROPOSED CONTROLLED SYSTEM

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The objective of the proposed optimal controller is to meet the requirements of QFT as follow:

1. To achieve a desirable performance by making the closed-loop response lie between the selected upper and lower tracking performance. The upper and lower tracking boundaries are known structure transfer functions and their parameters can be set according to the required specifications. The lower and upper tracking bounds are selected as :

$$T_u(s) = \frac{(4s+4)}{(s^2+4s+4)} \quad (12)$$

$$T_L(s) = \frac{16}{(s^2+8s+16)} \quad (13)$$

The specifications of the upper tracking are rise time equal to 0.36, settling time equal to 2.6 and overshoot equal to 13.5%, and the specifications of the lower tracking are rise time equal to 0.8, settling time equal to 1.4 and overshoot equal to zero.

2. Achieve robust stability by minimizing the maximum magnitude of output closed-loop system denote as δ_c.
It is subject to the following constrain:

$$\delta_c(jw) < \delta_d(jw) \quad (14)$$

where $\delta_d The$ difference between the upper and lower tracking is given as follow:

$$\delta_d(jw) = |T_u(jw)| - |T_L(jw)| \quad (15)$$

To make sure that robust stability was achieved the following equation must be achieved.

$$\left|\frac{f(jw)G_c(jw)p(jw)}{1+G_c(jw)p(jw)}\right| < \mu \quad (16)$$

3. Sensitivity function *S(jw)* will minimize to achieve a good disturbance rejection.
The constraints for quantitative feedback theory can be expressed as follow:

$$J = \left|\frac{f(jw)G_c(jw)p(jw)}{1+G_c(jw)p(jw)}\right| + \left|\frac{1}{1+G_c(jw)p(jw)}\right| \quad (17)$$

The first term of the cost function represents the magnitude of complement sensitivity, and the second term represents the magnitude of sensitivity.
where $f(jw)$ is prefilter, $G_c(jw) is$ controller and $p(jw)$ plant

In order to obtain the cost function for robust stability indices at all designed frequencies, the QFT technique requires the generation of robust system stability linked to the given robust stability specifications and the number of plant uncertainties. Then, these numerical bounds can be used directly in an automated design with the capability and flexibility of the algorithm.

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The desired frequencies are selected as follow:

W=[10^{-1}, 10^{0}, 10^{1}, 10^{2}]

Steps of design QFT using black hole optimization can preview as follow:

1. Define the plant of the system.

2. Define upper and lower tracks $T_u(t)$ and $T_L(t)$.

3. Transform time domain to frequency domain $T_u(s)$ *and* $T_L(s)$.

4. Calculate δ_d *(jw)*.

5. Controller and prefilter must be defined as in equations (10) and (11).

6. Initial population generates random it is select in this paper as 50.

7. Number of iteration is selected as 35.

8. For each population, calculate the cost function as in equation (17) then,

9. Compare between the values obtains, and the best value is *xi*.

10. Update position as in equation (8).

11. When iteration reach final value and quantitative constrain achieved then,

12. Stop program, and the array *F(xi)* contains the parameters of the controller and prefilter.

The resulting parametrs yield the following PID controller:

$$G_c(s) = 40 + \frac{44}{s} + 56s \tag{18}$$

Moreover, obtain prefilter as following:

$$f(s) = \frac{1}{(0.28s+1)(0.01s+1)} \tag{19}$$

where $f(s)$ is prefilter.

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VI. RESULTS AND DISCUSSION

As shown in *Fig*. 4, The response of the system goes immediately into infinity. It is obvious from this Figure that the system in the open-loop is unstable, allowing the ball to roll right off the end of the beam.

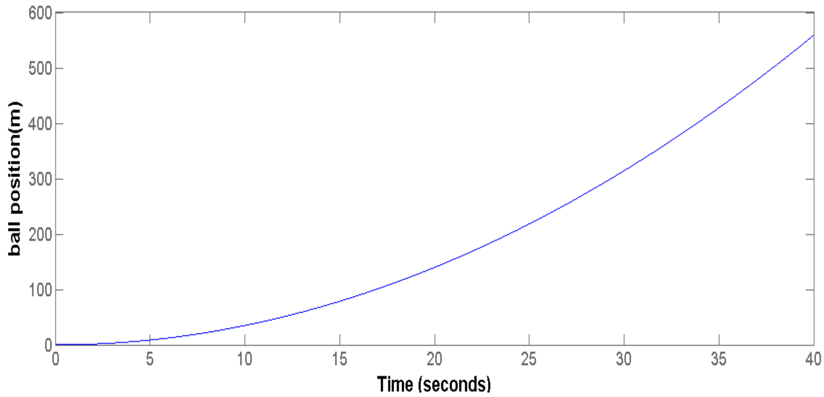

FIG. 4. TIME RESPONSE OF THE OPEN-LOOP OF BALL AND BEAM SYSTEM

After quantitative design controller by black hole optimization where BHO tunes the parameters of the controller, the value of parameters are obtained as follow $k_p = 40,\ k_i = 44,\ k_d = 56$, Desired performance of closed-loop response of uncertain parameters has been achieved as shown in *Fig.* 5, where the output of closed-loop system lies between the upper and lower boundaries.

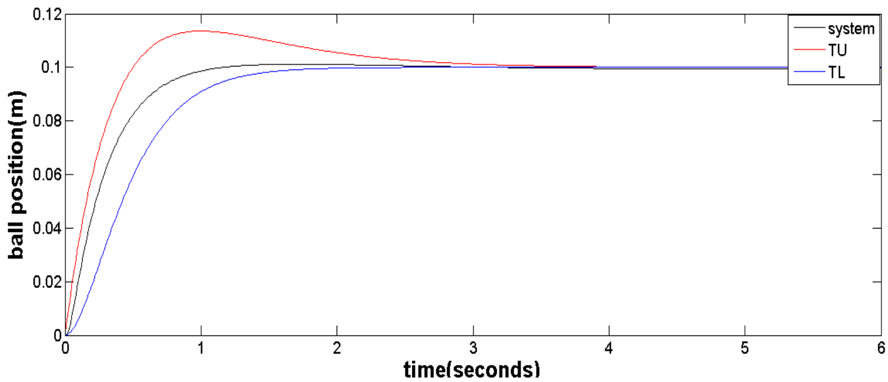

FIG. 5. TIME RESPONSE SPECIFICATIONS OF THE BALL AND BEAM SYSTEM

The specifications requirements are achieved, and a good performance is satisfied, as shown in *Fig.* 6. It is shown that the time response obtained by BHO based QFT. The minimum time settling is equal to 0.9 seconds, rise time equal to 0.5 seconds and overshoot equal to zero.

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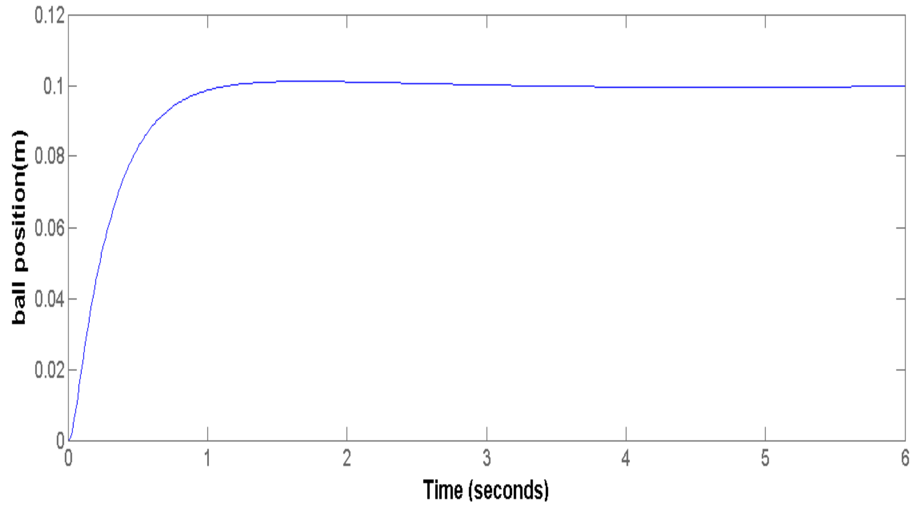

FIG. 6. TIME RESPONSE OF BALL AND BEAM SYSTEM

The closed loop response lies between lower and upper bound that mean the constrain is achieved and the $\delta_c(jw)$ has been reduced to be smaller than $\delta_d(jw)$, as shown in *Fig. 7.*

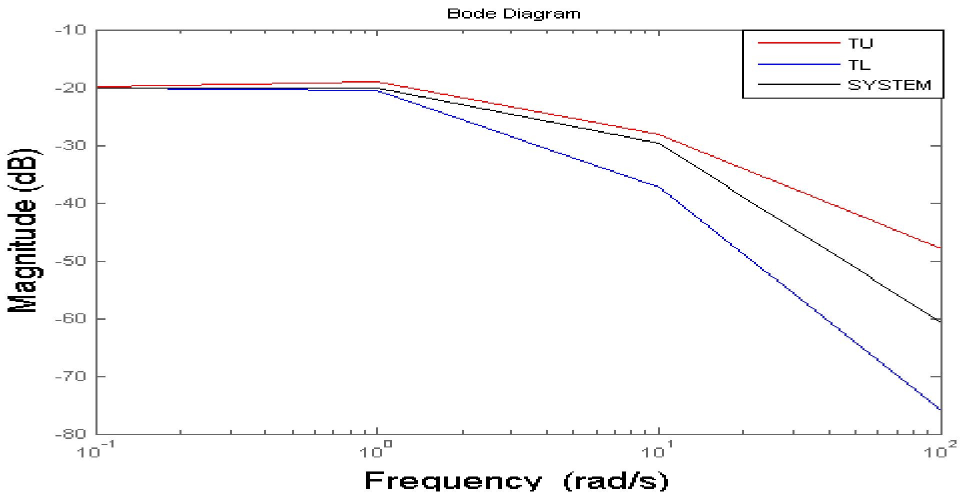

FIG. 7. CLOSED-LOOP FREQUENCY RESPONSE FOR SYSTEM

The closed-loop time response of an uncertain system lies between upper and lower tracking that mean the specifications desired is achieved, as shown in *Fig*. 8.

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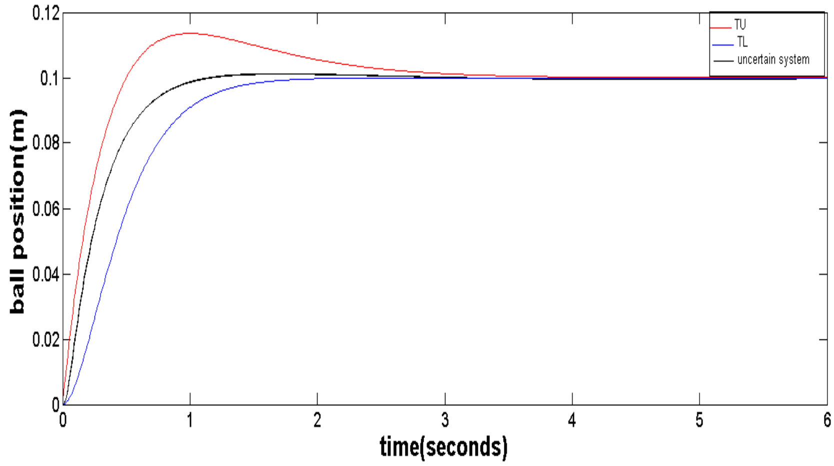

FIG. 8. CLOSED-LOOP TIME RESPONSE FOR UNCERTAIN SYSTEM

The closed-loop frequencies response of uncertain system lies between upper and lower tracking, which means the system's requirement specifications are achieved, as shown in *Fig*. 9.

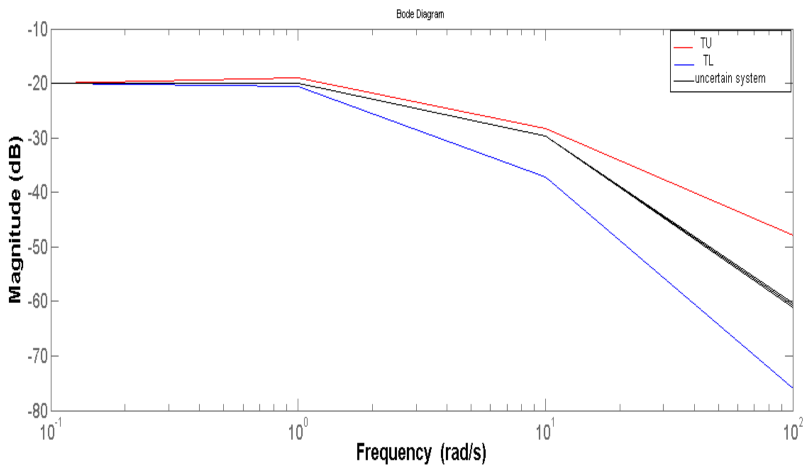

FIG. 9. CLOSED-LOOP FREQUENCY RESPONSE FOR UNCERTAIN BALL AND BEAM SYSTEM

VII. CONCLUSION

In this work, the design of quantitative PID controller for the ball and beam system has been presented. The BHO was used to automate the loop shaping procedure and optimize the design. The PID controller parameters have been obtained using BHO subject to the quantitative feedback theory constraints. The results showed that the proposed controller has achieved a more desirable performance for the ball and beam system.

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